

## **RATIONAL EXPECTATIONS AND THE THEORY OF ECONOMIC POLICY\***

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There is no longer any serious debate about whether monetary policy should be conducted according to rules or discretion. Quite appropriately, it is widely agreed that monetary policy should obey a rule, that is, a schedule expressing the setting of the monetary authority's instrument (e.g. the money supply) as a function of all the information it has received up through the current moment. Such a rule has the happy characteristic that in any given set of circumstances, the optimal setting for policy is unique. If by remote chance, the same circumstances should prevail at two different dates, the appropriate settings for monetary policy would be identical.

The central practical issue separating monetarists from Keynesians is the appropriate form of the monetary policy rule. Milton Friedman has long advocated that the monetary authority adopt a simple rule having no feedback from current and past variables to the money supply. He recommends that the authority cause the money supply to grow at some rate of  $x$  percent per year without exception. In particular, the Fed ought not to try to 'lean against the wind' in an effort to attenuate the business cycle.

\*This paper is intended as a popular summary of some recent work on rational expectations and macroeconomic policy and was originally prepared for a conference on that topic at the Federal Reserve Bank of Minneapolis in October 1974. The paper was previously published as paper 2 of the *Studies in Monetary Economics* series of the Federal Reserve Bank of Minneapolis. To make the main points simple, the paper illustrates things by using simple ad hoc, linear models. However, the ideas cannot really be captured fully within this restricted framework. The main ideas we are summarizing are due to Robert E. Lucas, Jr., and were advanced by him most elegantly in the context of a stochastic general equilibrium model [see Lucas (1972a)]. Lucas's paper analyzes policy questions in what we regard to be the proper way, namely, in the context of a consistent general equilibrium model. The present paper is a popularization that fails to indicate how Lucas's neutrality propositions are derived from a consistent general equilibrium model with optimizing agents. It is easy to overturn the 'neutrality' results that we derive below from an ad hoc structure by making ad hoc changes in that structure. The advantage of Lucas's model is that ad hoceries are given much less of a role and, consequently, the neutrality proposition he obtains is seen to be a consequence of individual agents' optimizing behavior. In summary, this paper is not intended to be a substitute for reading the primary sources, mainly Lucas (1972a, 1972b, 1973, forthcoming).

Within the context of macroeconomic models as they are usually manipulated, Friedman's advocacy of a rule without feedback seems indefensible. For example, suppose that a variable  $y_t$ , which the authority is interested in controlling, is described by the stochastic difference equation,

$$y_t = \alpha + \lambda y_{t-1} + \beta m_t + u_t, \quad (1)$$

where  $u_t$  is a serially independent, identically distributed random variable with variance  $\sigma_u^2$  and mean zero;  $m_t$  is the rate of growth of the money supply; and  $\alpha$ ,  $\lambda$  and  $\beta$  are parameters. The variable  $y_t$  can be thought of as the unemployment rate or the deviation of real GNP from 'potential' GNP. This equation should be thought of as the reduced form of a simple econometric model.

Suppose that the monetary authority desires to set  $m_t$  in order to minimize the variance over time of  $y_t$  around some desired level  $y^*$ . It accomplishes this by appropriately choosing the parameters  $g_0$  and  $g_1$  in the feedback rule,

$$m_t = g_0 + g_1 y_{t-1}. \quad (2)$$

Substituting for  $m_t$  from (2) into (1) gives

$$y_t = (\alpha + \beta g_0) + (\lambda + \beta g_1) y_{t-1} + u_t. \quad (3)$$

From this equation the steady-state mean of  $y$  is given by

$$E(y) = (\alpha + \beta g_0) / [1 - (\lambda + \beta g_1)], \quad (4)$$

which should be equated to  $y^*$  in order to minimize the variance of  $y$  around  $y^*$ . From (3) the steady-state variance of  $y$  around its mean (and hence around  $y^*$ ) is given by

$$\text{var } y = (\lambda + \beta g_1)^2 \text{var } y + \sigma_u^2$$

or

$$\text{var } y = \sigma_u^2 / [1 - (\lambda + \beta g_1)^2] \quad (5)$$

The monetary authority chooses  $g_1$  to minimize the variance of  $y$ , then chooses  $g_0$  from eq. (4) to equate  $E(y)$  to  $y^*$ . From eq. (5), the variance of  $y$  is minimized by setting  $\lambda + \beta g_1 = 0$ , so that  $g_1$  equals  $-\lambda/\beta$ . Then from eq. (4) it follows that the optimal setting of  $g_0$  is  $g_0 = (y^* - \alpha)/\beta$ . So the optimal feedback rule for  $m_t$  is

$$m_t = (y^* - \alpha)/\beta - (\lambda/\beta) y_{t-1}. \quad (6)$$

Substituting this control rule into (1) gives

$$y_t = y^* + u_t,$$

which shows that application of the rule sets  $y_t$  equal to  $y^*$  plus an irreducible noise. Notice that application of the rule eliminates all serial correlation in  $y$ , since this is the way to minimize the variance of  $y$ . Use of rule (6) means that the authority always expects to be on target, since its forecast of  $y_t$  at time  $t-1$  is

$$\hat{y}_t = \alpha + \lambda y_{t-1} + \beta m_t,$$

which under rule (6) equals  $y^*$ .

Friedman's  $x$ -percent growth rule in effect sets  $g_1$  equal to zero. So long as  $\lambda$  is not zero, that rule is inferior to the feedback rule (6).

This example illustrates all of the elements of the usual proof that Friedman's simple  $x$ -percent growth rule is suboptimal. Its logic carries over to larger stochastic difference equation models, ones with many more equations and with many more lags. It also applies where criterion functions have more variables. The basic idea is that where the effects of shocks to a goal variable (like GNP) display a stable pattern of persistence (serial correlation), and hence are predictable, the authority can improve the behavior of the goal variable by inducing offsetting movements in its instruments.

The notion that the economy can be described by presumably a large system of stochastic difference equations with fixed parameters underlies the standard Keynesian objections to the morism of monetarists who argue that the monetary authority should ignore other variables such as interest rates and concentrate on keeping the money supply on a steady growth path. The view that, on the contrary, the monetary authority should 'look at (and respond to) everything,' including interest rates, rests on the following propositions:<sup>1</sup> (a) the economic structure is characterized by extensive simultaneity, so that shocks that impinge on one variable, e.g. an interest rate, impinge also on most others; (b) due to lags in the system, the effects of shocks on the endogenous variables are distributed over time, and so are serially correlated and therefore somewhat predictable; and (c) the 'structure' of these lags is constant over time and does not depend on how the monetary authority is behaving. These propositions imply that variables that the authority observes very frequently, e.g. daily, such as interest rates, carry information useful for revising its forecasts of future value of variables that it can't observe as often, such as GNP and unemployment. This follows because the same shocks are affecting both the observed and the unobserved variables, and because those shocks have effects that persist. It follows then from (c) that the monetary authority should in general revise

<sup>1</sup>See Kareken, Muench, and Wallace (1973) for a detailed presentation of this view.

its planned setting for its policy instruments each time it receives some new and surprising reading on a variable that is determined simultaneously with a variable, like GNP or unemployment, that it is interested in controlling. Such an argument eschewing a simple  $x$ -percent growth rate rule in favor of 'looking at everything' has been made by Samuelson (1970):

'... when I learned that I has been wrong in my beliefs about how fast  $M$  was growing from December, 1968 to April, 1969, this news was just one of twenty interesting items that had come to my knowledge that week. And it only slightly increased my forecast for the strength of aggregate demand at the present time. That was because my forecasts, so to speak, do not involve "action at a distance" but are loose Markov processes in which a broad vector of current variables specify the "phase space" out of which tomorrow's vector develops. (In short, I knowingly commit that most atrocious of sins in the penal code of the monetarists – I pay a great deal of attention to all dimensions of "credit conditions" rather than keeping my eye on the solely important variable  $\dot{M}/M$ .)

... often, I believe, the prudent man or prudent committee can look ahead six months to a year and with some confidence predict that the economy will be in other than an average or "ergodic" state. Unless this assertion of mine can be demolished, the case for a fixed growth rate for  $M$ , or for confining  $M$  to narrow channels around such a rate, melts away.

These general presumptions arise out of what we know about plausible models of economics and about the findings of historical experience.'<sup>2</sup>

There can be little doubt about the inferiority of an  $x$ -percent growth rule for the money supply in a system satisfying proportions (a), (b), and (c) above. A reasonable disagreement with the 'look at everything, respond to everything' view would seemingly have to stem from a disbelief of one of those three premises. In particular, proposition (c) asserting the invariance of lag structures with respect to changes in the way policy is conducted would probably not be believed by an advocate of a rule without feedback.

Thus, returning to our simple example, a critical aspect of the proof of the suboptimality of Friedman's rule is clearly the assumption that the parameters  $\alpha$ ,  $\lambda$ , and  $\beta$  of the reduced form (1) are independent of the settings for  $g_0$  and  $g_1$  in the feedback rule. Macroeconometric models are almost always manipulated under such an assumption. However, Lucas (forthcoming) has forcefully argued

<sup>2</sup>Perhaps the 'look at everything' view goes some way toward rationalizing the common view that policy ought not to be made by following a feedback rule derived from an explicit, empirically estimated macroeconometric model. It might be argued that the models that have been estimated omit some of the endogenous variables that carry information about the shocks impinging on the system as a whole. If the authority has in mind an a priori model that assigns those variables an important role, it is appropriate for it to alter its policy settings in response to new information about those variables. Perhaps this is what some people mean by 'discretion,' although we aren't sure.

that the assumption is inappropriate, and that the parameters of estimated reduced forms like (1) in part reflect the policy responses in operation during the periods over which they are estimated. This happens because in the reduced forms are embedded the responses of expectations to the way policy is formed. Changes in the way policy is made then ought not to leave the parameters of estimated reduced forms unchanged.

To illustrate this point while continuing with our example, suppose that our reduced form (1) has been estimated during some sample period and suppose that it comes from the 'structure:'

$$y_t = \xi_0 + \xi_1(m_t - E_{t-1}m_t) + \xi_2y_{t-1} + u_t, \quad (7)$$

$$m_t = g_0 + g_1y_{t-1} + \varepsilon_t, \quad (8)$$

$$E_{t-1}m_t = g_0 + g_1y_{t-1}. \quad (9)$$

Here  $\xi_0$ ,  $\xi_1$ , and  $\xi_2$  are fixed parameters;  $\varepsilon_t$  is a serially independent random term with mean zero. We assume that it is statistically independent of  $u_t$ . Eq. (8) governed the money supply during the estimation period. The variable  $E_{t-1}m_t$  is the public's expectation of  $m_t$  as of time  $t-1$ . According to (9), the public knows the monetary authority's feedback rule and takes this into account in forming its expectations. According to eq. (7), unanticipated movements in the money supply cause movements in  $y$ , but anticipated movements do not. The above structure can be written in the reduced form

$$y_t = (\xi_0 - \xi_1g_0) + (\xi_2 - \xi_1g_1)y_{t-1} + \xi_1m_t + u_t, \quad (10)$$

which is in the form of (1) with  $\alpha = (\xi_0 - \xi_1g_0)$ ,  $\lambda = (\xi_2 - \xi_1g_1)$  and  $\beta = \xi_1$ . While the form of (10) is identical with that of (1), the coefficients of (10) are clearly functions of the control parameters, the  $g$ 's, that were in effect during the estimation period.

Suppose now that the monetary authority desires to design a feedback rule to minimize the variance of  $y$  around  $y^*$  under the assumption that the public will know the rule it is using and so use the currently prevailing  $g$ 's in (8) in forming its expectations, rather than the old  $g$ 's that held during the estimation period. The public would presumably know the  $g$ 's if the monetary authority were to announce them. Failing that, the public might be able to infer the  $g$ 's from the observed behavior of the money supply and other variables. In any case, on the assumption that the public knows what  $g$ 's the authority is using,  $\alpha$  and  $\lambda$  of eq. (1) come to depend on the authority's choice of  $g$ 's. This fundamentally alters the preceding analysis, as can be seen by substituting  $g_0 + g_1y_{t-1}$  for  $m_t$  in (10) to arrive at

$$y_t = (\xi_0 - \xi_1g_0) + (\xi_2 - \xi_1g_1)y_{t-1} + \xi_1(g_0 + g_1y_{t-1}) + u_t$$

or

$$y_t = \xi_0 + \xi_2 y_{t-1} + u_t + \xi_1 \varepsilon_t. \quad (11)$$

According to (11), the stochastic process for  $y_t$  does not even involve the parameters  $g_0$  and  $g_1$ . Under different values of  $g_0$  and  $g_1$ , the public's method of forming its expectations is also different, implying differences in the values of  $\alpha$  and  $\lambda$  in (1) under different policy regimes. In our hypothetical model, the resulting differences in  $\alpha$  and  $\lambda$  just offset the differences in  $g_0$  and  $g_1$ , leaving the behavior of  $y$  identical as a result. Put somewhat differently, our old rule 'set  $g_1 = -\lambda/\beta$ ' can no longer be fulfilled. For on the assumption that the public uses the correct  $g$ 's in forming its expectations, it implies

$$g_1 = -\lambda/\beta = (\xi_1 g_1 - \xi_2)/\xi_1 = g_1 - \xi_2/\xi_1$$

or

$$0 = -\xi_2/\xi_1,$$

which is an equality not involving the  $g$ 's, and one that the monetary authority is powerless to achieve. The rule ' $g_1 = -\lambda/\beta$ ' in no way restricts  $g_1$ .

The point is that estimated reduced forms like (1) or (10) often have parameters that depend partly on the way unobservable expectations of the public are correlated with the variables on the right side of the equation, which in turn depends on the public's perception of how policymakers are behaving. If the public's perceptions are accurate, then the way in which its expectations are formed will change whenever policy changes, which will lead to changes in the parameters  $\alpha$  and  $\lambda$  of the reduced-form equation. It is consequently improper to manipulate that reduced form as if its parameters were invariant with respect to changes in  $g_0$  and  $g_1$ . According to this argument, then, the above 'proof' of the inferiority of a rule without feedback is fallacious. The argument for the 'look at everything, respond to everything' view is correspondingly vitiated.

The simple model above is one in which there is no scope for the authority to conduct countercyclical policy by suitably choosing  $g_0$  and  $g_1$  so as to minimize the variance of  $y$ . Indeed, one choice of the  $g$ 's is as good as another, so far as concerns the variance of  $y$ , so that the authority might as well set  $g_1$  equal to zero, thereby following a rule without feedback. It seems, then, that our example contains the ingredients for constructing a more general defense of rules without feedback. These ingredients are two: first, the authority's instrument appears in the reduced form for the real variable  $y$  only as the discrepancy of the instrument's setting from the public's prior expectation of that setting; and second, the public's psychological expectation of the setting for the instrument equals the objective mathematical expectation conditioned on data available when the expectation was formed. The first property in part reflects a homogeneity of degree zero of supply with respect to prices and expected prices,

the natural unemployment rate hypothesis. But it also derives partly from the second property, which is the specification that the public's expectations are 'rational,' that is, are formed using the appropriate data and objective probability distributions.

The natural rate hypothesis posits that fully anticipated increases in prices have no effects on the rate of real economic activity, as indexed for example by the unemployment rate. A Phillips Curve that obeys the natural rate hypothesis can be written as

$$p_t - p_{t-1} = \phi_0 + \phi_1 U_t + {}_{t-1}p_t^* - p_{t-1} + \varepsilon_t, \quad \phi_1 < 0, \quad (12)$$

or

$$p_t - {}_{t-1}p_t^* = \phi_0 + \phi_1 U_t + \varepsilon_t, \quad (13)$$

where  $U_t$  is the unemployment rate,  $p_t$  is the log of the price level,  ${}_{t-1}p_t^*$  is the log of the price level that the public expects to prevail at time  $t$  as of time  $t-1$ , and  $\varepsilon_t$  is a random term. According to (12), the Phillips Curve shifts up by the full amount of any increase in expected inflation. That implies, as indicated by eq. (13), that if inflation is fully anticipated, so that  $p_t = {}_{t-1}p_t^*$ , then the unemployment rate is unaffected by the rate of inflation, since (13) becomes one equation,

$$0 = \phi_0 + \phi_1 U_t + \varepsilon_t,$$

that is capable of determining the unemployment rate independently of the rate of inflation.

As Phelps (1972) and Hall (forthcoming) have pointed out, in and of itself, the natural rate hypothesis does not weaken the logical foundations for 'activist' Keynesian macroeconomic policy, i.e. rules with feedback. This fact has prompted some to view the natural rate hypothesis as an intellectual curiosity, having but remote policy implications.<sup>3</sup> To illustrate, we complete the model by adding to (13) a reduced form aggregate demand schedule and an hypothesis about the formation of expectations. We subsume 'Okun's Law' in the former and assume it takes the form

$$p_t = am_t + bx_t + cU_t, \quad c > 0, \quad (14)$$

where  $m_t$  is the log of the money supply, the authority's instrument; and  $x_t$  is a vector of exogenous variables that follows the Markov scheme  $x_t = \delta x_{t-1} + u_t$ ,  $u_t$  being a vector of random variables. For price expectations, we posit the

<sup>3</sup>For example, see the remarks attributed to Franco Modigliani in *Brookings Papers on Economic Activity* 2 (1973, p. 480).

ad hoc, in general 'irrational' scheme,

$${}_{t-1}p_t^* = \lambda p_{t-1}, \quad (15)$$

where  $\lambda$  is a parameter. Using (13)–(15), we can easily solve for unemployment as a function of  $m_t$  and  $x_t$ ,

$$U_t = [\phi_0 - a(m_t - \lambda m_{t-1}) - b(x_t - \lambda x_{t-1}) + c\lambda U_{t-1} + \varepsilon_t] / (c - \phi_1). \quad (16)$$

It follows that the current setting for  $m_t$  affects both current and future values of unemployment and inflation. Given that the authority wishes to minimize a loss function that depends on current and future unemployment and perhaps inflation, the choice of  $m_t$  is a nontrivial dynamic optimization problem, the solution to which can often be characterized as a control rule with feedback. The optimal policy rule will depend on all of the parameters of the model and on the parameters of the authority's loss function. The policy problem in this context has been studied by Hall and Phelps. The authority can improve the characteristics of the fluctuations in unemployment and inflation by setting  $m$  so as to offset disturbances to the  $x$ 's.

In this system, if the authority has a 'humane' loss function that assigns regret to unemployment and that discounts the future somewhat, the authority should to some extent exploit the tradeoff between inflation and unemployment implied by (14) and (16). As Hall (forthcoming) has emphasized, the authority is able to do this by fooling people:

'... the benefits of inflation derive from the use of expansionary policy to trick economic agents into behaving in socially preferable ways even though their behavior is not in their own interests. ... The gap between actual and expected inflation measures the extent of the trickery. ... The optimal policy is not nearly as expansionary when expectations adjust rapidly, and most of the effect of an inflationary policy is dissipated in costly anticipated inflation.'

Hall has pinpointed the source of the authority's power to manipulate the economy. This can be seen by noting that removing the assumption that the authority can systematically trick the public eliminates the implication that there is an exploitable tradeoff between inflation and unemployment in *any* sense pertinent for making policy. The assumption that the public's expectations are 'rational' and so equal to objective mathematical expectations accomplishes precisely this. Imposing rationality amounts to discarding (15) and replacing it with

$${}_{t-1}p_t^* = E_{t-1}p_t, \quad (17)$$

where  $E_{t-1}$  is the mathematical expectation operator conditional on information



known at the end of period  $t-1$ . If (17) is used in place of (15), eq. (16) must be replaced with <sup>4</sup>

$$U_t = [a(m_t - E_{t-1}m_t) + b(x_t - E_{t-1}x_t) - \varepsilon_t]/(\phi_1 - c) - \phi_0/\phi_1. \quad (18)$$

To solve the model for  $U_t$ , it is necessary to specify how the authority is behaving. Suppose we assume that the authority uses the feedback rule,

$$m_t = G\theta_{t-1} + \eta_t, \quad (19)$$

where  $\theta_{t-1}$  is a set of observations on variables dated  $t-1$  and earlier, and  $\eta_t$  is a serially uncorrelated error term obeying  $E[\eta_t|\theta_{t-1}] = 0$ ;  $G$  is a vector conformable with  $\theta_{t-1}$ .

If the rule is (19) and expectations about  $m$  are rational, then

$$E_{t-1}m_t \equiv Em_t|\theta_{t-1} = G\theta_{t-1}, \quad (20)$$

since  $E[\eta_t|\theta_{t-1}] = 0$ . So we have

$$m_t - E_{t-1}m_t = \eta_t. \quad (21)$$

Substituting from (21) into (18) we have

$$U_t = [a\eta_t + b(x_t - E_{t-1}x_t) - \varepsilon_t]/(\phi_1 - c) - \phi_0/\phi_1. \quad (22)$$

Since the parameters  $G$  of the feedback rule don't appear in (22), we can conclude that the probability distribution of unemployment is independent of the values chosen for  $G$ . The distribution of the random, unpredictable component of  $m$ , which is  $\eta$ , influences the distribution of unemployment but there is no way in which this fact provides any logical basis for employing a rule with feedback. The  $\eta$ 's have a place in (22) only because they are unpredictable noise. On the basis of the information in  $\theta_{t-1}$ , there is no way that the  $\eta$ 's can be predicted, either by the authority or the public.

In this system, there is no sense in which the authority has the option to conduct countercyclical policy. To exploit the Phillips Curve, it must somehow

<sup>4</sup>Using (17), compute  $E_{t-1}$  of both sides of (13) and subtract the result from (13) to get

$$p_{t-t-1}p_t^* = \phi_1(U_t - E_{t-1}U_t) + \varepsilon_t. \quad (i)$$

Perform the same operation on (14) to get

$$p_{t-t-1}p_t^* = a(m_t - E_{t-1}m_t) + b(x_t - E_{t-1}x_t) + c(U_t - E_{t-1}U_t). \quad (ii)$$

Solve (i) for  $(U_t - E_{t-1}U_t)$  and substitute the result into (ii) to get

$$(1 - c/\phi_1)(p_{t-t-1}p_t^*) = a(m_t - E_{t-1}m_t) - (c/\phi_1)\varepsilon_t + b(x_t - E_{t-1}x_t). \quad (iii)$$

Upon substituting the implied expression for  $(p_{t-t-1}p_t^*)$  into (13), we get (18).

trick the public. But by virtue of the assumption that expectations are rational, there is no feedback rule that the authority can employ and expect to be able systematically to fool the public. This means that the authority cannot expect to exploit the Phillips Curve even for one period. Thus, combining the natural rate hypothesis with the assumption that expectations are rational transforms the former from a curiosity with perhaps remote policy implications into an hypothesis with immediate and drastic implications about the feasibility of pursuing countercyclical policy.<sup>5</sup>

As indicated above, by a countercyclical policy we mean a rule with feedback from current and past economic variables to the authority's instrument, as in a regime in which the authority 'leans against the wind.' While the present model suggests reasons for questioning even the possibility of a successful countercyclical policy aimed at improving the behavior of the unemployment rate or some closely related index of aggregate activity, the model is compatible with the view that there is an optimal rule for the monetary authority, albeit one that need incorporate no feedback. Such an optimal rule could be determined by an analysis that determines the optimal rate of expected inflation, along the lines of Bailey (1956) or Tobin (1967). If there is an optimal expected rate of inflation, it seems to imply restrictions on the constant and trend terms (and maybe the coefficients on some slowly moving exogenous variables like the labor force) of a rule for the money supply, but is not a cause for arguing for a feedback rule from endogenous variables to the money supply. The optimal rate of inflation, if there is one, thus has virtually no implications for the question of countercyclical policy. Furthermore, there is hardly any theoretical agreement about what the optimal rate of expected inflation is, so that it seems to be a weak reed for a control rule to lean on.

The simple models utilized above illustrate the implications of imposing the natural rate and rational expectations hypotheses in interpreting the statistical correlations summarized by the reduced forms of macroeconomic models, reduced forms that capture the correlations between monetary and fiscal variables on the one hand, and various real variables on the other hand. What is there to recommend these two hypotheses? Ordinarily, we impose two requirements on an economic model: first that it be consistent with the theoretical core of economics-optimizing behavior within a coherent general equilibrium framework; and second, that it not be refuted by observations. Empirical studies have not turned up much evidence that would cause rejection at high confidence levels of models incorporating our two hypotheses.<sup>6</sup> Furthermore, models

<sup>5</sup>The original version of such a 'neutrality' result is due to Lucas (1972a). His formulation is much deeper and more elegant than the one here, since his procedure is to start from individual agents' objectives and their information and then to investigate the characteristics of general equilibria. Less elegant formulations of neutrality results are in Sargent (1973) and Sargent and Wallace (1975).

<sup>6</sup>See Lucas (1973) and Sargent (1976, forthcoming) for empirical tests of the natural rate hypothesis.

along these lines seem to be the only existing ones consistent with individuals' maximizing behavior that are capable of rationalizing certain important correlations, such as the Phillips Curve, that exist in the data and are summarized by the reduced forms of macroeconomic models. The key feature of models that imply our hypotheses has been described by Lucas (1973): 'All formulations of the natural rate theory postulate rational agents, whose decisions depend on *relative* prices only, placed in an economic setting in which they cannot distinguish relative from general price movements.' Their inability separately to identify relative and overall nominal price changes is what gives rise to reduced forms like (1). But their rationality implies that only the surprise components of the aggregate demand variables enter. And this has the far reaching policy implications described above.

Several reasons can be given for using the hypotheses of rational expectations. An important one is that it is consistent with the findings that large parts of macroeconomic models typically fail tests for structural change (essentially versions of Chow tests).<sup>7</sup> As eq. (10) illustrates, if expectations are rational and properly take into account the way policy instruments and other exogenous variables evolve, the coefficients in certain representations of the model (e.g. reduced forms) will change whenever the processes governing those policy instruments and exogenous variables change. A major impetus to work on rational expectations is thus that it offers one reason, but probably not the only reason, that macroeconomic models fail tests for structural change. Indeed, the hypothesis of rational expectations even offers some hope for explaining how certain representations of the model change out of the sample.

A second reason for employing the hypothesis of rational expectations is that in estimating econometric models it is a source of identifying restrictions. The usual method of modelling expectations in macroeconomic models – via a distributed lag on the own variable – leaves it impossible to sort out the scalar multiplying the public's expectations from the magnitude of the weights in the distributed lag on own lags by which expectations are assumed to be formed. Therefore, the coefficients on expectations are generally underidentified econometrically. The way out of this has usually been to impose a unit sum on the distributed lag where expectations are formed. The problem is that this is an ad hoc identifying restriction with no economic reason to recommend it. It is generally incompatible with the hypothesis of rational expectations, which can be used to supply an alternative identifying restriction.<sup>8</sup>

A third reason for using the rational expectations hypothesis is that it accords with the economist's usual practice of assuming that people behave in their own best interests. This is not to deny that some people are irrational and neurotic. But we have no reason to believe that those irrationalities cause *systematic*

<sup>7</sup>For example, see Muench et al. (1974).

<sup>8</sup>See Lucas (1972b, 1973) and Sargent (1971).

and predictable deviations from rational behavior that a macroeconomist can model and tell the monetary authority how to compensate for. In this regard, it should be noted that the rational expectations hypothesis does not require that people's expectations equal conditional mathematical expectations, only that they equal conditional mathematical expectations plus what may be a very large random term (random with respect to the conditioning information). Thus we need only assume, for example, that

$${}_{t-1}p_t^* = E_{t-1}p_t + \phi_t, \quad (18a)$$

where  $E_{t-1}\phi_t = 0$ , and  $\phi_t$  is a random 'mother-in-law' term allowing for what may be very large random deviations from rationality. It is easy to verify that all of our results about countercyclical policy go through when (18a) is assumed. Therefore, in the context of the natural rate hypothesis, random deviations from perfectly rational expectations buy the monetary authority no leverage in making countercyclical policy. To be able to conduct a countercyclical policy, there must be systematic deviations from rational expectations which the monetary authority somehow knows about and can predict.

A fourth reason for adopting the hypothesis of rational expectations is the value of the questions it forces us to face. We must specify exactly the horizon over which the expectations are cast and what variables people are assumed to see and when, things that most macroeconomic models are silent on. In doing policy analysis under rational expectations, we must specify whether a given movement in a policy variable was foreseen beforehand or unforeseen, an old and important distinction in economics, but one that makes no difference in the usual evaluations of policy made with macroeconomic models.

Although the imposition of the natural rate and rational expectations hypotheses on reduced-form equations like (1) has allowed us to state some important results, such reasoning is no substitute for analysis of the underlying microeconomic models. Manipulation of such reduced forms even under the interpretation given by eqs. (7)–(9), which imposes the natural rate and rational expectations hypotheses, can be misleading because it leaves implicit some of the dependencies between parameters and rules. (For example, the 'structure' consisting of (7)–(9) is itself a reduced form suggested by Lucas (1973), some of whose parameters depend on the variance of  $\varepsilon_t$  in (8).) Also, a welfare analysis using such a model can be misleading because it requires adoption of an ad hoc welfare criterion, like the 'humane' loss function described above. In general, such a loss function is inconsistent with the usual welfare criterion employed in models with optimizing agents – Pareto optimality.

Finally, we want to take note of a very general implication of rationality that seems to present a dilemma. Dynamic models that invoke rational expectations can be solved only by attributing to the agents whose behavior is being described a way of forming views about the dynamic processes governing the

policy variables. Might it not be reasonable at times to attribute to them a systematically incorrect view? Thus suppose an economy has been operating under one rule for a long time when secretly a new rule is adopted. It would seem that people would learn the new rule only gradually as they acquired data and that they would for some time make what from the viewpoint of the policy-maker are forecastable prediction errors. During this time, a new rule could be affecting real variables.

A telling objection to this line of argument is that new rules are not adopted in a vacuum. Something would cause the change – a change in administrations, new appointments, and so on. Moreover, if rational agents live in a world in which rules can be and are changed, their behavior should take into account such possibilities and should depend on the process generating the rule changes. But invoking this kind of complete rationality seems to rule out normative economics completely by, in effect, ruling out freedom for the policymaker. For in a model with completely rational expectations, including a rich enough description of policy, it seems impossible to define a sense in which there is any scope for discussing the optimal design of policy rules. That is because the equilibrium values of the endogenous variables already reflect, in the proper way, the parameters describing the authorities' prospective subsequent behavior, including the probability that this or that proposal for reforming policy will be adopted.

Thus, suppose that a policy variable  $x_t$  is described by the objective probability distribution function,

$$\text{Prob}[x_{t+1} < F | Y_t, Z_t] = G[F, Y_t, Z_t; g_1, \dots, g_p], \quad (23)$$

where  $Y_t = [y_t, y_{t-1}, \dots]$  is a set of observations on current and past values of an endogenous variable, or vector of endogenous variables  $y$ ; and where  $Z_t = [z_t, z_{t-1}, \dots]$  is a set of observations on current and past values of a list of  $n$  exogenous variables and disturbances  $z_t^i$ ,  $i = 1, \dots, n$ . The probability distribution has  $p$  parameters  $g_1, \dots, g_p$ .

The probability distribution in (23) represents a very general description of the prospects about policy. It obviously can describe a situation in which policy is governed by a deterministic feedback rule, in which case the probability distribution collapses to a trivial one. The probability distribution in (23) can also model the case in which the monetary authority follows a feedback rule with random coefficients, coefficients that themselves obey some probability law. This situation is relevant where the monetary authority might consider changing the feedback rule from time to time for one reason or another. The probability distribution (23) can also model the case in which policy is in part simply random. The parameters  $[g_1, \dots, g_p]$  determine the probability function (23) and summarize all of the factors making up the objective prospects for policy. Policy settings appear to be random drawings from the distribution given in (23).

Now consider a rational expectations, structural model for  $y_t$  leading to a reduced form,

$$y_t = h(x_t, x_{t-1}, \dots, Z_t, E_t y_{t+1}), \quad (24)$$

where  $E_t y_{t+1}$  is the objective expectation of  $y_{t+1}$  conditioned on information observed up through time  $t$ . The  $Z_t$ 's are assumed to obey some probability distribution functions,

$$\begin{aligned} \text{Prob}[z_{t+1}^1 < H^1, z_{t+1}^2 < H^2, \dots, z_{t+1}^n < H^n | Z_t] \\ = F[H^1, H^2, \dots, H^n, Z_t]. \end{aligned}$$

A final form solution for the model is represented by an equation of the form

$$y_t = \phi(x_t, x_{t-1}, \dots, Z_t; \bar{g}), \quad (25)$$

with the property that

$$E_t y_{t+1} = \iint \phi(x_{t+1}, x_t, \dots, Z_{t+1}; \bar{g}) dG dF,$$

so that the expectation of  $y_{t+1}$  equals the prediction from the final form. The parameters  $\bar{g} = [g_1, \dots, g_p]$  turn out to be parameters of the final form (25), which our notation is intended to emphasize. Those parameters make their appearance in (25) via the process of eliminating  $E_t y_{t+1}$  from (24) by expressing it in terms of the  $x$ 's and  $Z$ 's. The parameters of  $F$  also are embedded in  $\phi$  for the same reason. That is, the function  $\phi$  must satisfy the equation

$$\begin{aligned} \phi(x_t, x_{t-1}, \dots, Z_t; \bar{g}) \\ = h[x_t, x_{t-1}, \dots, Z_t, \iint \phi(x_{t+1}, x_t, \dots, Z_{t+1}; \bar{g}) dG dF], \end{aligned}$$

in which the parameters of  $F$  and  $G$  make their appearance by virtue of the integration with respect to  $G$  and  $F$ .

The final form (25) formally resembles the final forms of the usual macroeconomic models without rational expectations. But there is a crucial difference, for in (25) there are no parameters that the authority is free to choose. The parameters in the vector  $\bar{g}$  describe the objective characteristics of the policy-making process and cannot be changed. They capture all of the factors that determine the prospects for policy. The authority in effect makes a random drawing of  $x$  from the distribution described by (23). The persons on the committee and staffs that constitute the authority 'matter' in the sense that they influence the prospects about policy and so are represented by elements of  $\bar{g}$ . But the authority has no freedom to influence the parameters of the final form (23), since the objective prospects that it will act wisely or foolishly are known to the public and are properly embedded in the final form (25).

The conundrum facing the economist can be put as follows. In order for a model to have normative implications, it must contain some parameters whose values can be chosen by the policymaker. But if these can be chosen, rational agents will not view them as fixed and will make use of schemes for predicting their values. If the economist models the economy taking these schemes into account, then those parameters become endogenous variables and no longer appear in the reduced-form equations for the other endogenous variables. If he models the economy without taking the schemes into account, he is not imposing rationality.

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